NON-HIERARCHICAL CONNECTIONS IN A TRANSPORTATIONAL AHP MODEL

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ABSTRACT
The aim of this paper is to introduce an Analytic Hierarchy Process (AHP) model on a public transportation system. The original AHP model is amended by non-hierarchical connections of system elements, which can be considered as extra information for the final decision. The model has been tested on a Japanese city bus transportation system. The created model is applicable for complex transport system development decisions. Based on our findings, ANP (which is recommended in MCDM literature for considering both hierarchical and non-hierarchical connections in a decision) could be substituted by an AHP-ISM approach, which is easier and more realistic to apply. Moreover, the connections of transport system elements are dominantly hierarchical, so AHP structure is advisable to keep; non-hierarchical linkages are just extra information for the decision. The created model not only promotes the AHP idea by considering non-hierarchical linkages but also offers a more practical decision making procedure than ANP.

Keywords: public transport, AHP, ANP, ISM, MCDM

INTRODUCTION
In multi-criteria decision making (MCDM) the structure of the examined elements plays a key role in the final scoring. There are two basic cases: hierarchical and network types are distinguished in scientific literature. For the determination of the structure, many authors suggest Interpretive Structural Modeling (ISM). Gorvett and Liu (2007) claimed that before setting up the decision model it is advisable to pursue an ISM procedure to shed light on the linkages among the relevant factors. Huang et al (2005) applied ISM for examining network impacts among decision elements. Thakkar et al (2008) suggested using ISM combined with graph theory for gaining an overall view of the structure of decision issues.

Provided the structure is hierarchical (based on ISM or graph theory or simply, on experts' consensus) and qualitative and quantitative factors are also included, Analytic Hierarchy Process (AHP) is an effective tool to apply for the multi-criteria decision problem (Saaty, 1977, Duleba et al, 2012, Carlsson and Walden, 1995, Yang and Shi, 2002).
In case the evaluators are facing a network type structure, Analytic Network Process (ANP) is recommended for solving the decision problem (Saaty, 2004, Ergu et al, 2011, Niemira and Saaty, 2004).

However it is stated in several operations research papers (e.g. Ergu et al, 2011) that by the application of ANP, the matrices to be evaluated are more complex and the evaluation procedure requires more effort from the decision makers.

This paper aims to create a different approach. The examined structure of the decision (weighting the necessity of element-development in a public transport system) is basically hierarchical, which has been proved by a conducted AHP research. In spite of that some non-hierarchical connections can be also detected, which are weaker than the dominant hierarchical ones. The objective is to consider these non-dominant linkages in the final weight score computation and to keep the original structure simultaneously. For this approach, ANP is not necessary to apply and AHP scoring can be kept.

THE AHP MODEL FOR PUBLIC TRANSPORT SUPPLY QUALITY

Having elaborated the relevant scientific references, the following hierarchical structure has been created for public bus transportation (Fig.1.):

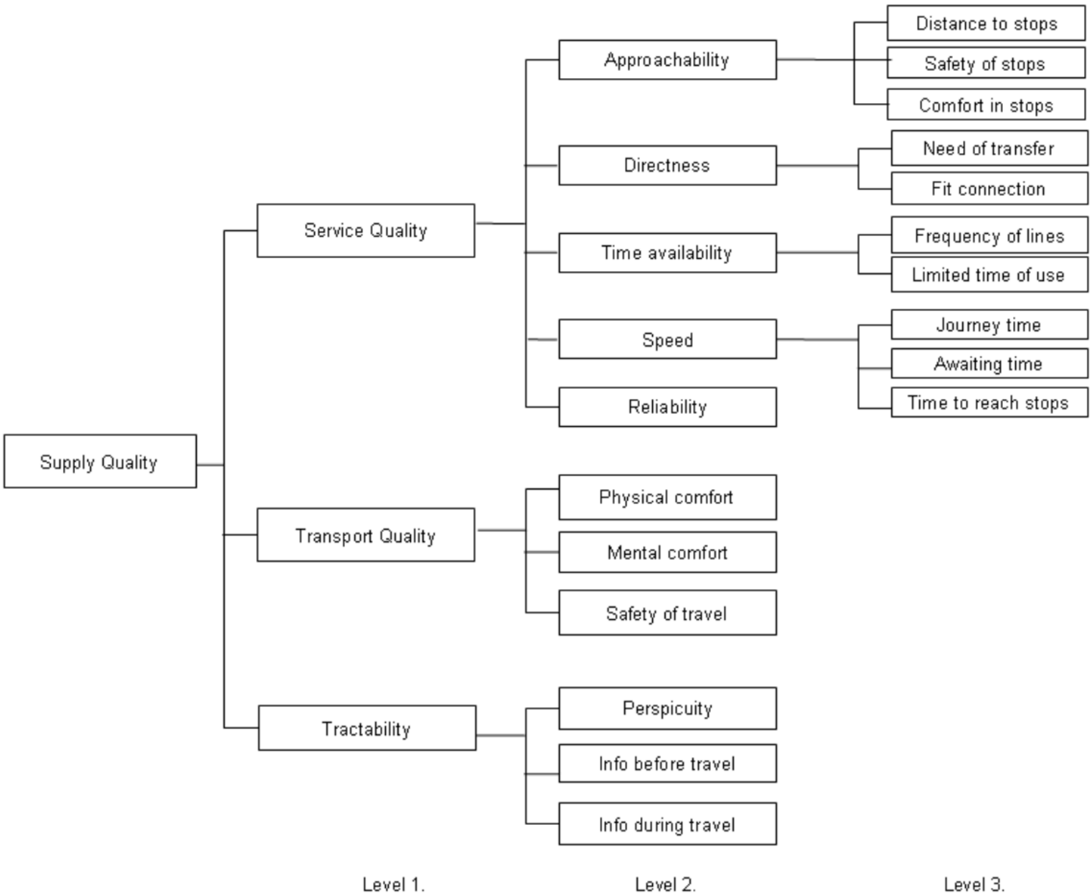

Figure 1 – The constructed AHP model

Then – following the AHP process – pair wise comparisons were made among the elements by constructing matrices within the groups of factors. After checking consistency, the individual evaluator scores were summarized by:

$$f(y_1,....,y_l) = \prod_{k=1}^{l} y_k^{\frac{1}{l}},\ l \geq 2, (y_1,....,y_l) \in I^l \quad (1)$$

Where "*f*" is the summarizing function, "l" is the number of the evaluators. y_k represents the proper indexed matrix element of the evaluator "k". I^l is the set of positive numbers.

In the next step, eigenvectors of the aggregated matrices were computed as:

$$w_{ei} = w_{ij} w_j \text{ , } i=1,\dots,n \quad (2)$$

w_j (j=1,…,m) normalized weight of the previous level

w_{ij} (i=1,…,n) normalized eigenvector-coordinate of the current level.

By this calculation, all weight scores could be determined for the structural elements. The higher the score has been, the more importance (in terms of its development) has been attained to the certain element. However this computational process considered only the hierarchical position and connections of each element.

ADDING NON-HIERARCHICAL LINKAGES TO THE MODEL

Evidently, the elements of Fig. 1. are not only hierarchically dependent. E.g. the factor of “frequency of lines” might have an impact on “physical comfort”, more frequently the buses are coming, less crowded is the vehicle. Another example is the effect of “frequency of lines” on “awaiting time”, etc.

Although these dependencies might be very important in terms of the decision making (the key question of this MCDM is: which element should be improved in order to gain the most improvement from passengers’ perspective), they are not considered in the AHP procedure.

For the consideration of these dependencies, external information is necessary from experts to nominate all non-hierarchical connections of the structural elements. Interpretive Structural Modeling (ISM) has been applied to highlight all relevant connections with the evaluation of 3 transportation experts.

For that objective, firstly the relation matrix has to be constructed. This is a binary and quadratic matrix (the number of rows and columns equals the number of structure elements), with the following principle:

$a_{ij} = 1$, if element “i” affects element “j”

$a_{ij} = 0$, otherwise

The general structure of a relation matrix (D):

Table 1 – The general structure of a relation matrix in ISM

	e_1	e_2	...	e_n
e_1	0	a_{12}	…	a_{1n}
e_2	a_{21}	0	...	a_{2n}
...	...	...	0	...
e_n	a_{n1}	a_{n2}	...	0

where

e_i is the i-th element of the system,

a_{ij} denotes the relation between i-th and j-th element.

Then the following step is to be taken:

RM=D+I (3)

so unity matrix (I) is added, which makes the main diagonal consist all 1-s. By that RM, reachability matrix is gained.

At this stage however, ISM procedure has been stopped. Following the original ISM rules, final RM should have been created in order to gain transitivity among the linkages and then iterations should have been done in order to create a structure based on the overall linkages of the elements. For this MCDM, only the direct and additional (so non-hierarchical) connections were to be determined, so the ISM procedure must be stopped.

Let us denote
e_i the i-th element of the system, i=1,…,n
k_{ei} all non-hierarchical connections of the element e_i, k=1,…,t,
μ_{kei} the specific rate of connection k_{ei}, which is determined by experts, $0<|\mu_{kei}|<1$.

μ_{kei} represents the rate of impact which is caused by improving the element e_i on the affected element. E.g. if “frequency of lines” is improved in the system, the element of “physical comfort” is also improved, but certainly not by 100%, only e.g. 20%. Then the rate of this connection for element e_i will be: 0,2, so μ_{kei}=0,2, if k=1 (in case it is the first non-hierarchical connection of “frequency of lines”).

Let us denote
e_{kei} the element which is affected by the element e_i in the connection of k_{ei}.
When μ_{kei}=0,2, then we can modify the original weight score of the element e_i with adding the score of the affected element e_{kei} multiplied by the rate of the connection, so with μ_{kei}.

$$w_{ei}' = w_{ei} + \sum_{k=1}^{t} \mu_{kei} w_{kei} \quad (4)$$

Where
w_{ei}' denotes the modified score of element e_i
w_{ei} the original score of element e_i
μ_{kei} the rate of (non-hierarchical) connection k
w_{kei} the score of the affected element by the impact of connection k.

w_{ei} and w_{kei} denote the original AHP scores of the elements obviously.

As can be seen, the AHP score of each element is modified by the non-hierarchical impacts on other elements in the structure. Those factors, which affect several other elements positively, will gain higher scores than the other ones. Note that μ_{kei} sometimes can be negative, in case one element affects the other negatively, in this case, the final score will be decreased for the specific factor. Based on (4) the importance and the influence of the factors within an arbitrary structure might be integrated in the decision making process.

CONCLUSION

Even in a dominantly hierarchical structure, some non-hierarchical connections can be determined, which might affect the final scoring in a multi-criteria decision making procedure. AHP process should not be replaced in those cases with the much more complicated ANP methodology; more proper way might be to modify the original AHP scores by adding the non-hierarchical impacts to the weight scores of elements.

For detecting all extra linkages, ISM is recommended but only till the phase of constructing the reachability matrix (RM).

The modification procedure will most likely cause rank-reversal of the elements, those factors which have many and strong impacts on others will gain higher weight scores in the final ranking of importance, and the others will probably be ranked behind than the original position.

The introduced procedure and approach might be even more important because in management practice, it is very hardly to find a clear, only hierarchically connected structure of elements.

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