MANAGING LATERAL TRANSSHIPMENTS IN A SUPPLY CHAIN ENVIRONMENT

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Introduction

With the intense competition in the wholesale business sector, there have been significant developments and applications of various supply chain management strategies to streamline the flow of goods. The stochastic nature of retailer demand makes precise demand forecasts impossible, warehouses in turn, cannot plan inventory easily. For cost competitiveness, warehouses tend to maintain the inventory at a low level and face the challenges of satisfying customer expectation for minimized storage cost. Eventhough cost competitiveness has become an imperative in inventory management,hence the adoption of this low invenotry strategy, when demand suddenly increases they need to backorder extra supplies from their suppliers or source extra from other wholesalers in the same region. The latter response is known as lateral transshipment which is faster but more expensive than sourcing from general suppliers. Inventory managers need to determine whether orders should be placed with a supplier or another wholesaler, and further, whether it should be in full or partly from a supplier and partly from another wholesaler. Such decisions are influenced by the requirement to minimize the total cost involved.

Previous research into wholesaler inventory management been problematic to adopt in real-world practice. There remains a need for simpler rules for lateral trans-shipment decisions. The method propsoed by this study extends the work done by Axsater (2003), who developed a number of important alternative decisions for the wholesaler to consider in lateral trans-shipments. Axsater (2003) put a significant focus on future cost difference for a certain initial state and the highly mathematical analysis and probability assumptions may not be easily understood by ordinary managers. We extend the objectives of Axsater (2003) but deviate from them in deriving a decision rule that can be conveniently used by inventory managers without performing complex mathematical calculations. Our approach handles the cost difference issue raised above using an alternative approach mainly based on predicted holding and backorder costs in different time periods.

The next section briefly review the relevant literature followed by the dvelopment of the proposed mathematical model for total costs, including purchasing costs, backordering costs and holding costs that are encountered in inventory replenishment by wholesalers and develops the decision rules for trans-shipments. Next we discuss the effectiveness of the decision model.

Literature Review

Optimization in the supply chain network environment has been studied from different perspectives including supplier selection (Ghodsypour and Brien,2001); determining transshipment quantities (Herer et al.,2006) delivery scheduling and distribution (Torabi et al.,2006) and product family selection (Lamothe et al.,2006). Taking into account the minimization of the total cost of logistics, including net price, storage, transportation and ordering costs, Ghodsypour and Brien (2001) studied the supplier selection problem in a multiple sourcing environment and proposed an algorithm comprising the pure non-linear programming model as a solution. On replenishment strategies and transhipments, Herer et al. (2006) focused on minimizing the expected long-run average cost including the replenishment, transshipment, holding, and penalty costs with the objective of finding the transshipment and replenishment quantities. They demonstrated how the values of the order-up-to quantities could be calculated using a sample-path-based optimization procedure and how to determine an optimal transshipment policy, using a linear programming/network flow framework. In another study, Lamothe et al. (2006) focused on minimizing the total cost comprising fixed cost of existence of physical items, fixed costs of existence of facilities, resource lines, shipping channel, and variable manufacturing, inventory and shipping costs and a mixed integer linear programming model was used as the solution.

Archibald et al. (1997) proposed using a stochastic dynamic program to optimize the decision of whether to laterally trans-ship. According to Axsäter (1990), when a wholesaler cannot supply goods to a retailer, lateral trans-shipment can take place and he proposed a method for optimizing the control policy of inventory replenishment. His model developed decision rules for lateral trans-shipments, aiming to evolve an integrated approach for supporting decisions regarding trans-shipment of goods. The proposed decision rule by Axsater (2003) is difficult to visualize the underlying concept and adopt practically. The evaluation of the decision rule suggested by Axsater (2003) lacks proper illustrations of how to put it in real practice. Consequently, the practical implications of that model are limited due to the requirement of sound knowledge of mathematics for application. Mathematical complexity of methods developed in previous studies hinder the application and adoption by logistics practitioners causing wholesaler inventory managers still using adhoc methods to make decisions. This paper studies the sourcing decisions of a wholesaler in fulfilling retailer demand and provides a pragmatic approach deriving simple decision rules that are conveniently adoptable by wholesaler inventory management.

Developing the Model

We assume that all wholesalers apply a periodic review policy described by Rosenshine and Obee [18] to replenish from external suppliers.

We define the following notation used in the model development.

W_i = the ith wholesaler,

N_i = the total number of suppliers to the wholesaler Wi,

S_{ij} = the jth supplier of the wholesaler Wi,

p_{ij} = the unit selling price by Sij to Wi ,

q_{ik} = the unit intra-shipment cost for Wi to intraship from Wk,

b_i = unit back-order cost at Wi per unit time,

h_i = unit holding cost for Wi per unit time,

t_0 = start of the scheduling period, t=0,

$g_{ij}(t)$ = delivery lead time probability mass function of Sij,

L_{ij} = lead time of Sij with duration equal to Lij times unit time interval,

L_{ij}^{max} = the maximal lead time of Sij,

$d_i(0)$ = the initial retailer demand at t=0 appearing at Wi,

$\lambda_i(t)$ = the retailer arrival intensity during the tth time interval at Wi,

$f^{n}_{i,m}$ = the probability of n retailers arriving at Wi with a total demand of m,

$\hat{d}_i(t)$ = the expected retailer demand at wholesaler Wi in the tth time interval,

$\hat{D}_{ij}$ = the expected retailer demand at wholesaler W_i over L^{max}_{ij}

Following Axsater [3], we assume retailer demand is a Compound Poisson process and retailers arrive at the wholesaler at an arrival intensity of λ. Assuming the probability of n number of retailers arriving at Wi wholesaler during a time interval of length t is $P_{d_{i,t}}(n)$,

$$P_{d_{i,t}}(n) = exp^{-\lambda_i t}\,\frac{\{\lambda_i\, t\}^n}{n!} \tag{1}$$

If the conditional probability of n number of retailers requires m demand is given by

$P_{d_{i,t}}(m|n) = f^{n}_{1,m}$ then the probability of retailer demands at the Wi wholesaler during the time interval t is,

$$P_{d_{i,t}}(m) = \sum_{n=1}^{m} P_{d_{i,t}}(m|n) P_{d_{i,t}}(n)$$

$$= \sum_{n=1}^{m} e^{-\lambda_i t} f^{n}_{1,m}(n)\,\frac{\{\lambda_i\, t\}^n}{n!} \tag{2}$$

The expected retailer demand at Wi wholesaler during a time t is given by

$$\hat{d}_i\,(t) = \sum_{m=1}^{\infty} m\, P_{d_{i,t}}\,(m)$$

Applying the function of $P_{d_{i,t}}\,(m)$ from equation 2,

$$\hat{d}_i\,(t) = exp^{-\lambda_i t} \sum_{m=1}^{+\infty} \sum_{n=1}^{m} \frac{\{\lambda_i\, t\}^n}{n!}\, m f^{n}_{1,m}(n) \tag{3}$$

At t=0, demand with the wholesaler Wi, $d_i(0)$ is the outstanding demand carried forward from the previous scheduling period and that needs to be urgently fulfilled.

If the supplier, Sij of the wholesaler Wi takes Lij lead time to deliver the order and assuming that the probability mass function of delivery lead time of Sij is gij(t) then the expected lead time of E(L_{ij}) is given by

$$E(L_{ij}) = \int_{t=0}^{L_{ij}} t\, g_{ij}\,(t)dt$$

Let's consider that one scheduling period is the maximum lead time of Sij, L_{ij}^{max}. Then the expected demand at Wi during one scheduling period is

$$\widehat{D}_{ij} = \sum_{k=1}^{L_{ij}^{max}} \hat{d}_i(k)$$

From equation 3,

$$\widehat{D}_{ij} = \sum_{k=1}^{L_{ij}^{max}} e^{-\lambda_i k} \sum_{m=1}^{+\infty} \sum_{n=1}^{m} \frac{\{\lambda_i k\}^n}{n!} m\, f_{1,m}^{n}(n) \qquad (4)$$

Time period	Expected demand
$[t_0 - t_1]$	$\hat{d}_i(1)$
$[t_1 - t_2]$	$\hat{d}_i(2) - \hat{d}_i(1)$
$[t_2 - t_3]$	$\hat{d}_i(3) - \hat{d}_i(2)$
…	…
$[(L_{ij}^{max} - 2) - (L_{ij}^{max} - 3)]$	$\hat{d}_i(L_{ij}^{max} - 1) - \hat{d}_i(L_{ij}^{max} - 2)$
$[(L_{ij}^{max} - 1) - (L_{ij}^{max})]$	$\hat{d}_i(L_{ij}^{max}) - \hat{d}_i(L_{ij}^{max} - 1)$

Table 1: Expected demand at the wholesaler Wi

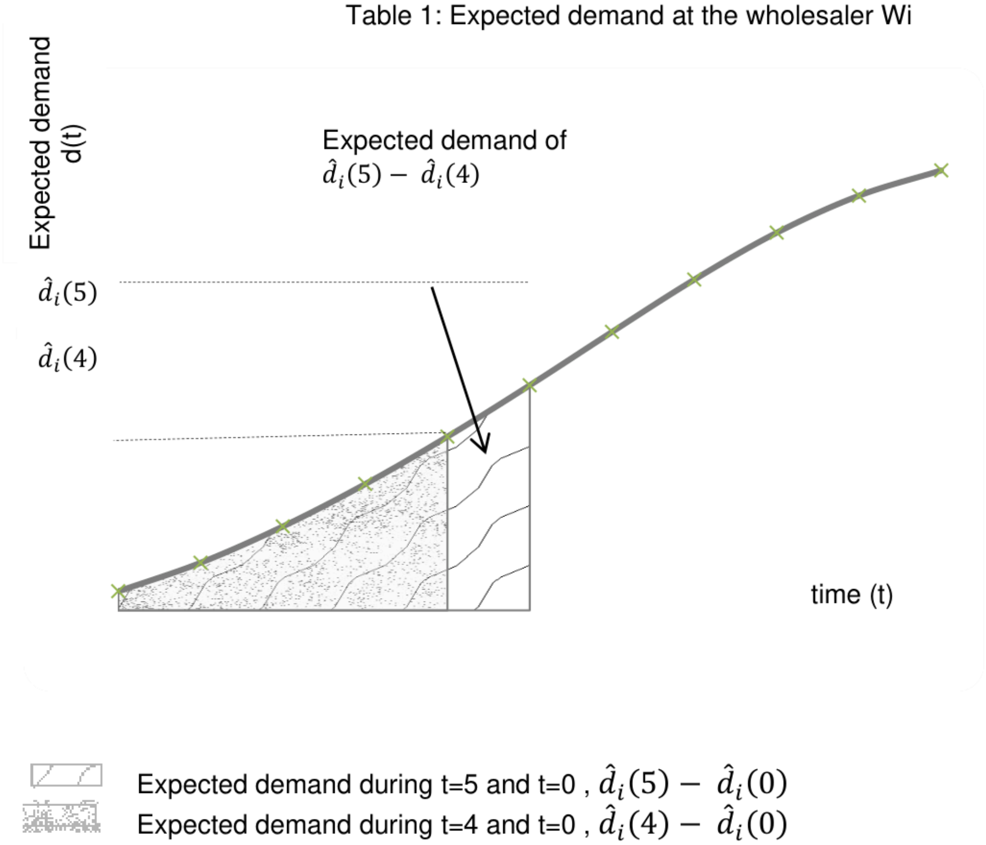

Figure 1: Diagram of expected demand at the wholesaler Wi

Figure 1 presents a schematic diagram of the expected retailer demand at wholesaler Wi and Table 1 presents the expected demand for each time period. The calculations in the following sections refer back to these for better understanding.

Costs of Supply

Inventory costs of a wholesaler consist of three components: purchasing costs of P_{ijk} of the orders to suppliers and intra-shipments from the other wholesalers, backordering costs of B_{ijk} for unfulfilled retailer demands and holding costs of H_{ijk} for carrying inventory for potential demands.

$$C_{ijk}(x) = P_{ijk}(x) + B_{ijk}(x) + H_{ijk}(x) \tag{5}$$

We consider the maximum lead time taken by the supplier is the scheduling period for the wholesaler's decision. In our model, the period to update the wholesaler is set to the maximal lead time of the given supplier. So all the orders made should be received in the period. Wi therefore there will be no outstanding orders from its suppliers at the initial point of a new period, which will simplify the total cost model and decision rules for intra-shipment. So the inventory level of the wholesaler is equal to its inventory position.

Purchasing costs

The purchasing cost has two components depending on the source of supply: supplier or another wholesaler. If the unit cost of purchasing from the supplier Sj is pij, the unit cost of trans-shipping from the wholesaler Wk is qik and the trans-shipped quantity is x units then the purchasing cost of $P_{ijk}(x)$ is given by

$$P_{ijk}(x) = p_{ij}\left(d_i(0) - l_i(0) + \widehat{D}_{ij} - x\right) + q_{ik}(x) \tag{6}$$

Where di(0) and li(0) are the outstanding order quantity and the inventory level of the Wi at t=0.

Backordering costs

For the initial back-ordered quantity of ($d_i(0) - \ell_i(0) - x$) the back-ordering time is the expected arrival time of order from the supplier. We assume that intra-shipping cost from another wholesaler is significantly higher than the purchasing cost from suppliers. Hence, only the urgent orders are intra-shipped and others are sourced from suppliers. We also assume that intra-shipped orders have zero lead time. Hence, at the beginning of the scheduling period, any outstanding order that cannot be fulfilled by the local inventory and intra-shipped orders, generate backordering costs, until the expected supplier order supply entry day. Subsequent retailer demands expected to arrive during the time range of 0≤t≤E(Lij)-1 are not fulfilled due to stock-out status and consequently they generate backordering cost depending on the back-order time.

Based on the expected retailer demand as shown in Figure 1 and Table 1, the following Table 2 presents the back-order time for the unfulfilled demand at the wholesaler, Wi.

Time period of demand	demand	Backorder period (e.g. number of days)
$at\ t_0$	$(d_i(0) - l_i(0) - x)$	$E(L_{ij})$
$[t_0 - t_1]$	$\hat{d}_i(1)$	$E(L_{ij}) - 1$
$[t_1 - t_2]$	$\hat{d}_i(2) - \hat{d}_i(1)$	$E(L_{ij}) - 2$
$[t_2 - t_3]$	$\hat{d}_i(3) - \hat{d}_i(2)$	$E(L_{ij}) - 3$
...	...	...
$[\{E(L_{ij}) - 2\} - \{E(L_{ij}) - 3\}]$	$\hat{d}_i(E(L_{ij}) - 2) - \hat{d}_i(E(L_{ij}) - 3)$	2
$[\{E(L_{ij}) - 1\} - \{E(L_{ij}) - 2\}]$	$\hat{d}_i(E(L_{ij}) - 1) - \hat{d}_i(E(L_{ij}) - 2)$	1

Table 2: Backorder time period for the unfulfilled demand

If Bijk is the backorder cost for the wholesaler Wi for the orders placed with the supplier Sij after the intrashipments from the Wk and bij is the unit back-order cost then ,

$$B_{ijk}(x) = b_{ij}(d_i(0) - l_i(0) - x)E(L_{ij}) + b_{ij}\int_{t=1}^{E(L_{ij})-1}\{\hat{d}(t) - \hat{d}(t-1)\}\{E(L_{ij}) - t\}\,dt \quad (7)$$

Holding costs

During the scheduling period, holding costs are incurred after the order is received at the expected lead time of the supplier and the outstanding demands are delivered i.e. during the time period of $E(L_{ij}) + 1 \leq t \leq L_{ij}^{max}$. Based on the expected demand as shown in Figure 1 and Table 1, the following table 3 presents the holding cost for the stock received from the supplier at $E(L_{ij})$.

Time period of demand	demand	Holding period (e.g. in number of days)
$\{E(L_{ij}) + 1\} - E(L_{ij})$	$\hat{d}_i(E(L_{ij}) + 1) - \hat{d}_i E(L_{ij})$	1
$\{E(L_{ij}) + 2\} - \{E(L_{ij}) + 1\}$	$\hat{d}_i(E(L_{ij}) + 2) - \hat{d}_i(E(L_{ij}) + 1)$	2
$\{E(L_{ij}) + 3\} - \{E(L_{ij}) + 2\}$	$\hat{d}_i(E(L_{ij}) + 3) - \hat{d}_i(E(L_{ij}) + 2)$	3
...	...	...
$\{L_{ij}^{max} - 1\} - \{L_{ij}^{max} - 2\}$	$\hat{d}_i(L_{ij}^{max} - 1) - \hat{d}_i(L_{ij}^{max} - 2)$	$E(L_{ij}^{max} - 1) - E(L_{ij})$
$L_{ij}^{max} - \{L_{ij}^{max} - 1\}$	$\hat{d}_i(L_{ij}^{max}) - \hat{d}_i(L_{ij}^{max} - 1)$	$E(L_{ij}^{max}) - E(L_{ij})$

Table 3: Holding period of the stock received from the supplier

If the total holding cost of H_{ijk} during the scheduling period and the unit holding cost of the Wi wholesaler is hi then,

$$H_{ijk}(x) = \int_{t=E(L_{ij})+1}^{L_{ij}^{max}} \{\hat{d}(t) - \hat{d}(t-1)\}\{t - E(L_{ij})\} h_{ij}\, dt \quad (8)$$

Decision rule for intra-shipment orders

By substituting the functions of purchasing costs, backordering costs and holding costs in equation 5, the total cost during the scheduling period is given by,

$$C_{ijk}(x) = p_{ij}\left(d_i(0) - l_i(0) + \widehat{D}_{ij} - x\right) + q_{ik}(x) + (d(0) - l(0) - x)EL_{ij}b_i + b_i \int_{t=0}^{E(L_{ij})-1} \hat{d}(t) - \hat{d}(t-1)\{E(L_{ij}) - t\}\, dt + \int_{t=E(L_{ij})+1}^{L_{ij}^{max}} \{\hat{d}(t) - \hat{d}(t-1)\}\{t - E(L_{ij})\} h_i\, dt$$

$$C_{ijk}(x) = \{-p_{ij} - E(L_{ij})b_{ij} + q_{ik}\}x + p_{ij}\left(d_i(0) - l_i(0) + \widehat{D}_{ij}\right) + \{d_i(0) - l_i(0)\}\, E(L_{ij})b_i + b_i \int_{t=1}^{E(L_{ij})-1} \{\hat{d}(t) - \hat{d}(t-1)\}\{E(L_{ij}) - t\}\, dt + \int_{t=E(L_{ij})+1}^{L_{ij}^{max}} \{\hat{d}(t) - \hat{d}(t-1)\}\{t - E(L_{ij})\} h_i\, dt \quad (9)$$

With respect to the quantity of trans-shipment (x), the total cost function as shown in equation 9 is linear. When the tangent of the above linear function ($-p_{ij} - E(L_{ij})b_i + q_{ik}$) is negative the total cost decreases with the increasing number of trans-shipments.

Hence the decision rule for intra-shipments is

$$-p_{ij} - E(L_{ij})b_i + q_{ik} < 0$$

$$q_{ik} < p_{ij} + E(L_{ij})b_i \quad (10)$$

If the above decision rule shown in equation 10 is satisfied, the higher the quantity of trans-shipment the lower the total cost for the wholesaler Wi.

Discussion and Conclusion

The decision method propsoed in this study assists the inventory management of wholesaler operations in making decisions on whether to trans-ship outstanding urgent retailer demands or back order from suppliers in full. The main benefit of this decision support system is the ease of application by wholesaler inventory management. The decision is driven by the important cost minimization objectives, the simplicity of the rules and the need for less cumbersome data inputs to the model, underpin the ease of adoption. The main decision rule needs only the unit purchasing cost from suppliers, unit trans-shipment cost from another wholesaler, own unit backordering cost, and the expected lead time from its suppliers. The proposed decision rule algorithm can be integrated as a module of the existing enterprise resource planning applications with the capability of extracting data from previous transaction records in the internal databases, thereby extending the capability of achieving more reliable and accurate expected costs in inventory management operations.

Reference

- Archibald, T.W., Sassen,S. A. E., Thomas, L.C., An Optimal Policy for a Two Depot Inventory Problem with Stock Transfer, Management Science 43 (1997) 173-183.
- Axsäter, S. Modelling emergency lateral transshipments in inventory systemss, Management Science 36 (1990) 1329-1338.

- Axsäter, Sven. (2003). A New Decision Rule for Lateral Transshipments in Inventory Systems. *Management Science, 49*(9), 1168-1179. doi: 10.2307/4134033
- Ghodsypour, S. H., & O'Brien, C. (2001). The total cost of logistics in supplier selection, under conditions of multiple sourcing, multiple criteria and capacity constraint. *International Journal of Production Economics, 73*(1), 15-27.
- Herer, Yale T., Tzur, Michal, & Yücesan, Enver. (2006). The multilocation transshipment problem. [doi: 10.1080/07408170500434539]. *IIE Transactions, 38*(3), 185-200. doi: 10.1080/07408170500434539
- Lamothe, Jacques, Hadj-Hamou, Khaled, & Aldanondo, Michel. (2006). An optimization model for selecting a product family and designing its supply chain. *European Journal of Operational Research, 169*(3), 1030-1047.
- Rosenshine, Matthew, & Obee, Duncan. (1976). Analysis of a Standing Order Inventory System with Emergency Orders. *Operations Research, 24*(6), 1143-1155. doi: 10.2307/169982
- Torabi, S. A., Fatemi Ghomi, S. M. T., & Karimi, B. (2006). A hybrid genetic algorithm for the finite horizon economic lot and delivery scheduling in supply chains. *European Journal of Operational Research, 173*(1), 173-189.